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Abstract

Sea ice can fail under lateral compression by buckling, a form of flexural motion, and this is thought to be a common phenomenon during pressure ridging. This type of motion is modeled as a stability problem for the vibration of a thin plate over water which is excited by ambient motions while the plate is under compression. The plate has an 'effective' bending modulus to account for actual depth varying modulus of elasticity of the ice. The dispersion relation for the flexural vibrations is evaluated, and explicit formulas are found for the critical value of compressive stress causing failure and for the most unstable wavelength. The critical compressive stress is about 10^6 N/m^2 for 10 cm thick first-year ice and slightly less than 10^7 N/m^2 for 3 m thick multi-year ice. An experimental technique for evaluating the effective bending modulus and the critical compressive stress is suggested, namely, by measurement of the dispersion curve for free flexural waves which are omnipresent in Arctic pack ice. Photographic evidence of pre-buckling flexure is provided for an example case on a refrozen lead.

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Introduction

Predictions for the bending and, ultimately, the failure of Arctic ice under load are important in a number of contexts, such as surface transportation, aircraft landings, and foundations for structures (Kerr 1976). Both vertical loads of objects placed on the ice and lateral loads from compression of the ice by forces from surrounding floes or engineering structures can cause the ice to bend and to fail, often spectacularly. The safety of personnel on the ice requires knowledge concerning its performance as a structural material, and there has been a great deal of theoretical and experimental work associated with static and dynamic loading of ice sheets to gain this knowledge.

The understanding of the deformation of an ice sheet under vertical forces has been well developed for both static loads (cf. Kerr 1976, Nevel 1977) and moving loads (cf. Eyre 1977, Beltaos 1981, DiMarco et al 1991), and the flexural rigidity of the ice is an important material parameter. However, the deformation under lateral compression, such as pressure imposed on the ice sheet by neighboring floes, has not been so well studied. In this case, the lateral force causes rafting and pressure ridging when it exceeds some unknown, but presumably critical, value. Under large scale compression, the ice typically fails by flexure or buckling (Parmenter and Coon 1972, Kerr 1978, 1980). This situation is directly related to bending of the ice plate, and the flexural rigidity parameter again is an important one.

Visual observations of thinner ice on refrozen leads exhibit a common occurrence of two specific phenomena, namely, finger rafting and corrugations in the ice. If the surrounding multi-year (and therefore thicker) ice floes have converged on the lead with little shear, they compress the younger ice until it buckles. The buckling is often associated with the ice forming a series of sinusoidal vertical deflections whose wavenumber is parallel to the direction of the compressive force. Sometimes, a number of wavelengths may be observed, while other times there may be only one or two. In our experience, all but one example of this phenomenon have been stationary ones, as the events usually happened sometime prior to the actual observations. The corrugations are generated, presumably due to compression of the sheet, but the ice did not buckle before the compressive motion was stopped. This possibly happens because the surrounding floes came into contact somewhere along their edges and the thicker ice of the multi-year floes took up the load, thus stopping the compression of the thinner ice in the refrozen lead. Then, because the new ice was rather thin when this occurred, it was held in this position as it quickly grew thicker with ongoing freezing, thus stabilizing the corrugated feature. The buckling event literally has been 'frozen in time'. Evidently, this is a reasonably common mechanism for generating these corrugations and buckling the ice. On the other hand, if the ice actually buckles, it then will raft or ridge as the convergence continues. The purpose of this paper is to present a simple theory for

the generation of these buckling corrugations prior to the occurrence of the failure of the ice, and to provide a visual example.

The threshold, or critical, value of the lateral compression load above which the plate will fail has been predicted theoretically by Kerr (1978), and confirmed experimentally in the laboratory for urea ice by Sodhi et al (1983). It is of great interest to examine the critical conditions for Arctic sea ice floes, and to have a convenient field method for determining the ice structural parameters which determine the critical stress.

In this paper, the theory of propagation of free flexural waves on the ice (treated as a thin, laterally homogeneous, elastic plate) is used to analyze the effect of lateral compressive forces. The dispersion relation for flexural waves predicts solutions having growing amplitudes when the compressive stress exceeds a critical value, and experimental techniques are used to determine the important ice parameters in a convenient manner. A theoretical model for this phenomenon is constructed in the next section, data for specific ice parameters and an example observation are exhibited in the following one, and we conclude with recommendations for future research.

Theory

The ice sheet is assumed to be homogeneous in both horizontal dimensions and to experience an external horizontal load which is uniform along one dimension. Thus, the response can be considered in a plane and, with depth-dependent modulus of elasticity, the governing equation is

$$\rho_i h\zeta_{tt} + hP\zeta_{xx} + D\zeta_{xxxx} = -\rho_w[\phi_t+g\zeta], \tag{1}$$

where

$$D = Eh^3/(12[1-\nu^2]), \tag{2}$$

ϕ is the velocity potential of motions in the water, E is an effective Young's modulus; D is the effective flexural rigidity, ν is the shear modulus, h is ice thickness, ζ is vertical deflection, P is the compressive force, g is gravity, and ρ_i and ρ_w are ice and water density, respectively. The first term on the left in Eqn 1 is the acceleration, and the remaining two terms on the left are the forces due to compression and elasticity of the sheet. The terms on the right are forces due to the presence of water supporting the sheet, with the first one being the dynamic pressure and the second one the hydrostatic force exerted by the water upon deflection from the equilibrium level. Ewing and Crary (1934) analyzed the case of a uniform ice plate on water and Mansfield (1989), for example, includes the effect of compression. The effect of Young's modulus varying with depth has been analyzed separately by Newman and Forray (1962) and Kerr and Palmer (1972). The effective value of the flexural rigidity is defined by the relation

$$D = \int E_{eff}(z)(z-z_o)^2(1-\nu^2)^{-2}\, dz \tag{3}$$

where z_o, the depth of the neutral plane, is given by

$$z_o = h - \int E_{eff}(z)(h-z)dz\, [\int E_{eff}(z)dz]^{-1}. \tag{4}$$

and all the integrals are evaluated over the depth h. Typical depth profiles of both first year and multi-year ice are given by DiMarco et al (1993), and a sample profile for multi-year ice is provided in Fig 1. The local values of Young's modulus were calculated from measurements of temperature, salinity, and brine volume, and using the empirical formula of Tucker et al (1989). As is clear from the plot, the top of the ice typically is much stronger because it is fresher and colder, so the neutral axis is above the middle of the sheet.

Periodic solutions of Eqn (1) of the form

$$\zeta(x,t) = e^{[i(kx-\omega t)]} \tag{5}$$

are assumed, and the eqn may be reduced to the algebraic form

$$\omega^2 = k^2[\rho_w g/k - Phk + Dk^3]/[\rho_w + \rho_i hk] \tag{6}$$

which relates the frequencies and wavenumbers of the allowable solutions of the unforced equations of motion. This equation is the dispersion relation for the waves and, for zero compressive stress, the free wave solutions have previously been considered in detail. These wave solutions are referred to as flexural-gravity waves, and they have been measured on both multi-year and first-year ice in the Arctic (cf. Dugan et al 1992 for a recent example and for references to earlier work). Fig 2 is a typical example of measurements for both the gravity and flexural limbs of the dispersion curve for naturally occurring waves on sea ice in the central Arctic. The curves representing the data have been obtained by calculation of the phase of the cross spectrum between the vertical motions as measured by accelerometers or geophones at two points separated by a fixed distance on the ice. Then, the wavenumber is reassembled from the known distance between the sensors and the phase lag. Finally, the calculated phase speed is the frequency divided by the measured wavenumber. The smooth curves in the plot are the theory for nominal multi-year ice parameters (as shown in Fig 1), and plotted in this figure as a function of the ice thickness.

In addition, solutions of Eqn (1) have been examined for vertical forces on the ice. Weights on the ice moving at constant speed (cf. Eyre 1977, Beltaos 1981) and also atmospheric pressure fluctuations moving along with the wind (DiMarco et al 1991) cause quasi-static deflections as long as the speed is slower than the minimum phase speed of the free flexural-gravity waves on the dispersion curve. This minimum speed is called the critical speed, and it can be calculated directly from Eqn (6). However, if the speed of the forcing function exceeds the critical speed, and the frequency-wavenumber bandwidth of the forcing function overlaps the

dispersion curve, inhomogeneous solutions of Eqn (1) exist as resonances which grow with time, and these solutions are not entirely represented by the form in Eqn (6). Thus, from previous research on forced motions, there is an expectation that a critical speed, that is the minimum speed of free waves, is central to understanding the motions of the ice.

As shown in Eqn 3 and 4 and Figs 1 and 2, this critical speed is a function of the flexural strength of the ice, i.e., both its thickness and effective Young's modulus. In addition, as shown in Eqn (6), the critical speed also is a function of the ambient compressive stress in the ice. The relative importance of the compression term is determined by the numerator in Eqn (6). As the compressive force increases, the frequency in Eqn (6) (and the corresponding wave speed) is reduced toward zero for wavenumbers near the critical value. For forces larger than the critical one, the right hand side of Eqn (6) becomes negative, and the frequencies in Eqn (5) are complex, and the instability grows with time (Timoshenko 1936). The equivalent critical force in the classical mechanics problem of buckling of an elastic column is commonly called the 'Timoshenko load'.

The value of the force at this critical threshold is obtained by setting the term in brackets in Eqn (6) to zero and solving for P, that is,

$$hP = Dk^2+\rho_w g/k^2. \tag{7}$$

The minimum value as a function of the wavenumber is

$$hP = 2[\rho_w gD]^{1/2}, \tag{8}$$

or

$$P = [Eh\rho_w g/(3(1-\nu^2))]^{1/2}. \tag{9}$$

Note that the critical force is proportional to the square root of both the ice thickness and the Young's modulus. The value of the wavenumber at this minimum point is

$$k = [\rho_w g/D]^{1/4}, \tag{10}$$

so the length of the most unstable wave is equal to 2π times the characteristic length of the ice sheet, which is such an important parameter for any bending problem. This result for the wavelength is not unexpected, as when the ice is compressed, the point which is most likely to flex is distant by a characteristic length. The plate cannot flex as easily on a smaller scale because of the stronger elastic forces, and not on longer scales because of the gravitational support of the water. Note that Eqn (9) predicts that the critical load is only dependent upon the square root of the ice thickness. Since the modulus also is weakly dependent upon the thickness, with thicker (and older) ice being stronger, the critical load actually increases at a rate that is closer to being linear in the ice thickness.

Liu and Mollo-Christensen (1988) have considered the effect of lateral compression on an ice sheet from a different point of view. They modeled the speed dependence of ocean swell traveling into the ice pack from the open ocean, in an attempt to understand an observation which was interpreted as focusing of wave energy. Lateral compression in the ice sheet was identified as a mechanism possibly explaining the observation of local accumulation of wave energy. The present theoretical formulation is similar to theirs, but the interpretation and use of the results are quite different. Because of energy losses in the ice sheet at frequencies for which the ice flexes, typical ocean swell does not propagate freely in pack ice except at frequencies below that of the minimum phase speed (cf. Wadhams 1973 or Dugan et al 1992). Thus, in realistic conditions, the compression effect is not expected to be as important for swell at extended distances into the pack ice as it is for the buckling problem through its effect on amplifying instabilities.

Now, for any specific ice plate, the Eqns (3) and (4) can be evaluated to estimate the effective rigidity, or Young's modulus. This can be accomplished by a number of means. Techniques have been crafted to estimate this by in situ bending of cantilever beams (Schwarz et al 1981, Tatinclaux and Hirayama 1982), and also by analyzing ice cores and comparing the measured temperature, salinity, and brine volume values with empirical correlations (cf. Cox and Weeks 1983, Mellor 1986, Tucker et al 1989). In addition, various geophysical techniques have been used in which the elastic properties have been obtained from measurement of the speed of propagating waves in ice samples in the laboratory (Boyle and Sproule 1931, Northwood 1947) and in the field (Hunkins 1960). The latter used flexural waves, and we elect this particular mode of propagation because these waves are solutions of the equations of motion discussed in the foregoing. DiMarco et al (1993) have used this particular method to compare results for the flexural rigidity between the geophysical method and the empirical method using ice cores.

Predictions and Observations

As a numerical example, the multi-year ice represented by the data in Figs 1 and 2 had 3.5 m thickness, and application of the geophysical method gave a result of 1.2 GN/m^2 for the effective Young's modulus and 3 GNm for the flexural rigidity. The predicted value of the buckling stress for this case is 3.7 MN/m^2 and the critical wavelength is 145 m.

In actuality, though, refrozen lead ice is of special interest because it is thinner than the multi-year ice and, by Eqn (9), is more likely to buckle in a given situation where a specific floe is compressed by neighboring multi-year floes. The temperature profile of first year ice is nearly linear from the freezing temperature at the base to air temperature at the surface. The Young's modulus profile therefore also is relatively linear with depth. In the following, we take values of parameters evaluated from field measurements provided by AARI scientists. Fig 3 is the effective modulus for several values of the surface temperature.

..... awaiting Smirnov's data for this plot .....

Using eqn (..) to calculate the effective E results in Fig 3. This can be entered into eqn (..) to get

an approximate eqn for the critical stress as a function of only the ice thickness and surface temperature, and the result is shown in Fig 4, with the accompanying wavelength in Fig 5.

An example of this phenomenon is provided in Fig 6 which contains photographs of a refrozen lead taken during the Soviet North Pole camp #???. Apparently, the thin ice in the lead between multi-year floes was compressed enough to cause the growth of the unstable waves, but the compressive stress was relieved before the ice actually buckled. The ice thickness was measured by drilling holes....

The air temperature was ...., so the effective rigidity was about...

The measured wavelength of the corrugations was....., and this compares with a value of .... calculated from the above theory.

As often occurs, the event which generated these waves was not observed, so the association with this instability is circumstantial. It is possible that it occurred a number of days earlier, so the ice actually would have been considerably thinner than the value that was measured at the time of the observations. If so, the value of the rigidity when the event occurred would have been considerably smaller, and the most unstable wavelength much shorter. In this sense, this example is only a one-sided bound on what happened during the event, and the observation only provides a (rather weak) consistency check on the theory.

Discussion and Conclusions

A model has been derived to estimate the critical lateral load that an ice sheet will sustain before it fails in a buckling mode. A crucial material parameter is the flexural rigidity, or the effective modulus of elasticity, and this may be estimated by using measurements of flexural waves which naturally occur in the pack ice (particularly during ridge-building events). This methodology represents a consistent estimate of the appropriate material parameter. The theory assumes that the ice sheet is homogeneous in the horizontal, but it may have vertical stratification of the elastic modulus. The sheet is compressed uniformly along the horizontal axis, and wavelike solutions are sought that satisfy the flexural-gravity wave equations. The theory predicts the critical value of compressive stress and force from measurements of the flexural wave speed at frequencies removed from the minimum speed. Using this technique, values of the critical stress and length of the most unstable wave are predicted for both first and multi-year ice. An example of corrugations observed on refrozen lead ice is shown to be consistent with the theory???

Finally, a future experiment is indicated, wherein the minimum in the dispersion curve would be measured by more detailed observations of the vibrations of the ice sheet. In the present model, the shape of the dispersion curve near this minimum provides a direct estimate of the state of compression of the ice, but no attempts at measuring this directly have been made to date. In addition, a controlled experiment using man-made vibrations would contribute to observations of naturally occurring ones on pack ice and nearby refrozen lead ice would provide data which is

much more appropriate for comparison with the model.

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References

Beltaos, S., Field studies on the response of floating ice sheets to moving loads, Can. J. Civ. Engng., 8, 1981, 1-8.

Boyle, R.W. and D.O. Sproule, Velocity of longitudinal vibration in solid rods (ultrasonic method) with special reference to the elasticity of ice, Can. J. of Research, 5, 1931, 601-618.

Cox, G.F.N. and W.F. Weeks, Equations for determining the gas and brine volumes in sea ice, J. Glaciol., 29, 1983, 306-316.

DiMarco, R.L., J.P. Dugan, and W.W. Martin, Ice motions forced by boundary layer turbulence, J. Geophys. Res. 96 (C6), 1991, 10617-10624.

DiMarco, R.L., J.P. Dugan, W.W. Martin, and W.J. Tucker, Sea ice flexural rigidity: a comparison of methods, Cold Regions Sci. and Technol., 21, 1993, 247-255.

Dugan, J.P., R.L. DiMarco, and W.W. Martin, Low-frequency vibrational motion of arctic pack ice, J. Geophys. Res., 97(C4), 1992, 5381-5388.

Dugan, J.P., Model spectrum for low frequency ice vibrations in the central Arctic. Ann. Rev. Fluid Mech., 27, 1995, 115-168.

Ewing, M. and A.P. Crary, Propagation of elastic waves in ice. Part II, Physics, 5, 1934, 181-184.

Eyre, D., The flexural motions of a floating ice sheet induced by moving vehicles, J. Glaciology, 19, 1977, 555-570.

Hunkins, K., Seismic studies of sea ice, J. Geophys. Res., 65, 1960, 3459-3472.

Kerr, A.D., and W.W. Palmer, The deformations and stresses in floating ice plates, Acta Mechanic, 15, 1972, 57-72.

Kerr, A.D., The bearing capacity of floating ice plates subjected to static or quasi-static loads, J. Glaciology, 17, 1976, 229-268.

Kerr, A.D., On the buckling force of floating ice plates, in Physics and Mechanics of Ice, P. Tryde, ed., Springer-Verlag, Berlin, 1980, 163-178.

Liu, A.K. and E. Mollo-Christensen, Wave propagation in a solid ice pack, J. Geophys. Res., 18, 1988, 1702-1712.

Mansfield, E.H., The Bending and Stretching of Plates, 2nd ed., Cambridge Univ. Press, N.Y., 1989.

Nevel, D.E., Concentrated loads on a floating ice sheet, J. Glaciology, 19, 1977, 237-245.

Newman, M. and M. Forray, Thermal stresses and deflections in thin plates with temperature dependent elastic modulus, J. Aerosp. Sci., 29(3), 1962, 372-373.

Northwood, T.D., Sonic determination of the elastic properties of ice, Can. J. Research, 25, 1947, 88-95.

Parmenter, R.R. and M.D. Coon, Model of pressure ridge formation in sea ice, J. Geophys. Res., 77, 1972, 6565-6575.

Sodhi, D.S., F.D. Hatnes, K. Kato, and K. Hirayama, Experimental determination of the buckling loads of floating ice sheets, Annals of Glaciology, 4, 1983, 260-265.

Timoshenko, S., Theory of Elastic Stability. McGraw-Hill, N.Y., 1936.

Tucker, W.B. III, J.A. Richter-Menge and A.J. Gow, Variations in mechanical properties within a multi-year ice floe, in Proc. IEEE Oceans '89, Seattle, 1989, 1287-1291.

Wadhams, P., Attenuation of swell by sea ice, J. Geophys. Res., 78, 1973, 3552-3563.

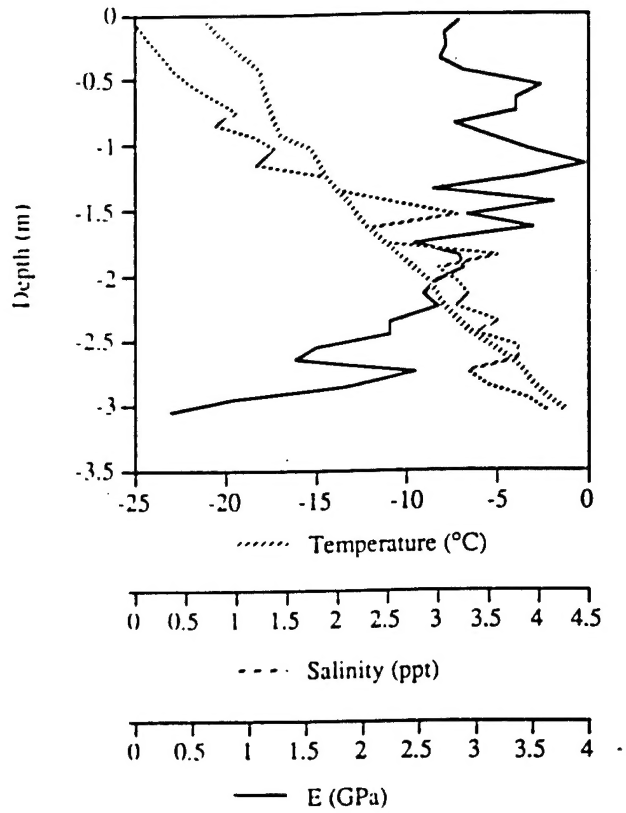

Figure 1. Typical vertical profiles of temperature, salinity and derived Yong's modulus from measurements on multi-year ice floe

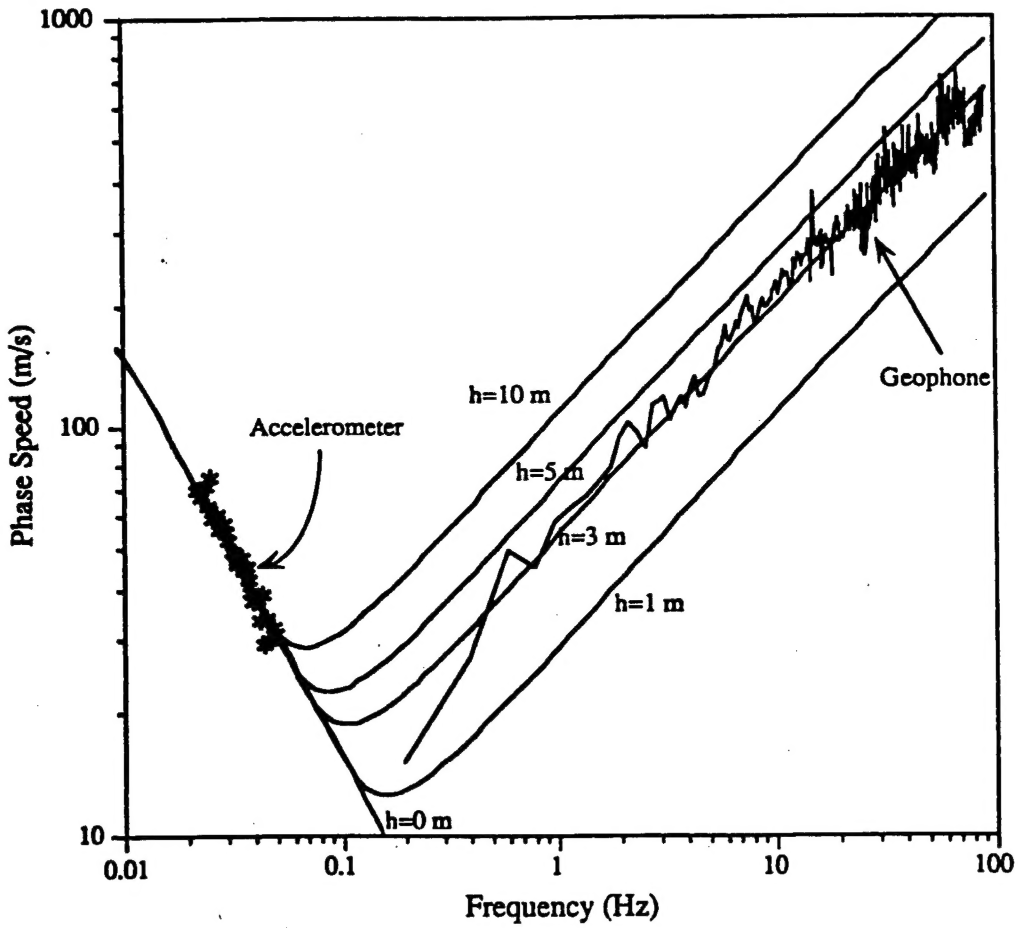

Figure 2. Dispersion relation for flexural-gravity waves on multi-year ice floe. Smooth curves are theory for non-resonent waves on ice of several indicated thicknesses with Young's modulus as given in Figure 1. Data are from accelerometer measurements on low frequency and geophones on high frequency for naturally occurring waves.